Chapter 1

Introduction

In this chapter we will introduce some basic terminology and lay the groundwork for the course. We will explain in general terms what statistics and probability are and the problems that these two areas of study are designed to solve.

1.1 Basic Definitions and Concepts

LEARNING OBJECTIVE

1. To learn the basic definitions used in statistics and some of its key concepts.

We begin with a simple example. There are millions of passenger automobiles in the United States. What is their average value? It is obviously impractical to attempt to solve this problem directly by assessing the value of every single car in the country, adding up all those numbers, and then dividing by however many numbers there are. Instead, the best we can do would be to estimate the average. One natural way to do so would be to randomly select *some* of the cars, say 200 of them, ascertain the value of each of those cars, and find the average of those 200 numbers. The set of all those millions of vehicles is called the *population* of interest, and the number attached to each one, its value, is a *measurement*. The average value is a *parameter*: a number that describes a characteristic of the population, in this case monetary worth. The set of 200 cars selected from the population is called a *sample*, and the 200 numbers, the monetary values of the cars we selected, are the *sample data*. The average of the data is called a *statistic*: a number calculated from the sample data. This example illustrates the meaning of the following definitions.

Definition

A **population**[1] *is any specific collection of objects of interest. A* **sample**[2] *is any subset or subcollection of the population, including the case that the sample consists of the whole population, in which case it is termed a* **census**.

Definition

A **measurement**[3] *is a number or attribute computed for each member of a population or of a sample. The measurements of sample elements are collectively called the* **sample data**[4].

[1] All objects of interest.

[2] The objects examined.

[3] A number or attribute computed for each member of a set of objects.

[4] The measurements from a sample.

Definition

A **parameter**[5] *is a number that summarizes some aspect of the population as a whole. A* **statistic**[6] *is a number computed from the sample data.*

Continuing with our example, if the average value of the cars in our sample was $8,357, then it seems reasonable to conclude that the average value of all cars is about $8,357. In reasoning this way we have drawn an inference about the *population* based on information obtained from the *sample*. In general, *statistics* is a study of data: describing properties of the data, which is called *descriptive statistics*, and drawing conclusions about a population of interest from information extracted from a sample, which is called *inferential statistics*. Computing the single number $8,357 to summarize the data was an operation of descriptive statistics; using it to make a statement about the population was an operation of inferential statistics.

Definition

Statistics[7] *is a collection of methods for collecting, displaying, analyzing, and drawing conclusions from data.*

Definition

Descriptive statistics[8] *is the branch of statistics that involves organizing, displaying, and describing data.*

[5] A number that summarizes some aspect of the population.

[6] A number computed from the sample data.

[7] Collection, display, analysis, and inference from data.

[8] The organization, display, and description of data.

Definition

Inferential statistics[9] *is the branch of statistics that involves drawing conclusions about a population based on information contained in a sample taken from that population.*

The measurement made on each element of a sample need not be numerical. In the case of automobiles, what is noted about each car could be its color, its make, its body type, and so on. Such data are *categorical* or *qualitative*, as opposed to *numerical* or *quantitative* data such as value or age. This is a general distinction.

Definition

Qualitative data[10] *are measurements for which there is no natural numerical scale, but which consist of attributes, labels, or other nonnumerical characteristics.*

Definition

Quantitative data[11] *are numerical measurements that arise from a natural numerical scale.*

Qualitative data can generate numerical sample statistics. In the automobile example, for instance, we might be interested in the proportion of all cars that are less than six years old. In our same sample of 200 cars we could note for each car whether it is less than six years old or not, which is a qualitative measurement. If 172 cars in the sample are less than six years old, which is 0.86 or 86%, then we would estimate the parameter of interest, the population proportion, to be about the same as the sample statistic, the sample proportion, that is, about 0.86.

The relationship between a population of interest and a sample drawn from that population is perhaps the most important concept in statistics, since everything

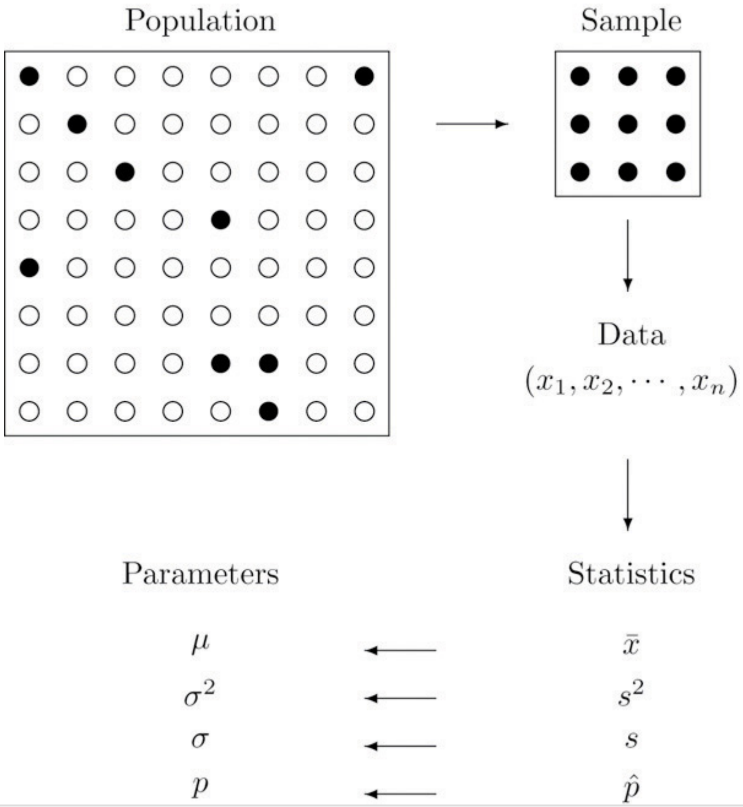
[9] Drawing conclusions about a population based on a sample.

[10] Measurements for which there is no natural numerical scale.

[11] Numerical measurements that arise from a natural numerical scale.

else rests on it. This relationship is illustrated graphically in Figure 1.1 "The Grand Picture of Statistics". The circles in the large box represent elements of the population. In the figure there was room for only a small number of them but in actual situations, like our automobile example, they could very well number in the millions. The solid black circles represent the elements of the population that are selected at random and that together form the sample. For each element of the sample there is a measurement of interest, denoted by a lower case *x* (which we have indexed as $x_1, \ldots, x_n$ to tell them apart); these measurements collectively form the sample data set. From the data we may calculate various statistics. To anticipate the notation that will be used later, we might compute the sample mean $\overline{x}$ and the sample proportion $\hat{p}$, and take them as approximations to the population mean μ (this is the lower case Greek letter mu, the traditional symbol for this parameter) and the population proportion *p*, respectively. The other symbols in the figure stand for other parameters and statistics that we will encounter.

Figure 1.1 *The Grand Picture of Statistics*



KEY TAKEAWAYS

- Statistics is a study of data: describing properties of data (descriptive statistics) and drawing conclusions about a population based on information in a sample (inferential statistics).
- The distinction between a population together with its parameters and a sample together with its statistics is a fundamental concept in inferential statistics.
- Information in a sample is used to make inferences about the population from which the sample was drawn.

EXERCISES

1. Explain what is meant by the term *population.*

2. Explain what is meant by the term *sample*.

3. Explain how a sample differs from a population.

4. Explain what is meant by the term *sample data.*

5. Explain what a *parameter* is.

6. Explain what a *statistic* is.

7. Give an example of a population and two different characteristics that may be of interest.

8. Describe the difference between *descriptive statistics* and *inferential statistics.* Illustrate with an example.

9. Identify each of the following data sets as either a population or a sample:

   a. The grade point averages (GPAs) of all students at a college.
   b. The GPAs of a randomly selected group of students on a college campus.
   c. The ages of the nine Supreme Court Justices of the United States on January 1, 1842.
   d. The gender of every second customer who enters a movie theater.
   e. The lengths of Atlantic croakers caught on a fishing trip to the beach.

10. Identify the following measures as either quantitative or qualitative:

    a. The 30 high-temperature readings of the last 30 days.
    b. The scores of 40 students on an English test.
    c. The blood types of 120 teachers in a middle school.
    d. The last four digits of social security numbers of all students in a class.
    e. The numbers on the jerseys of 53 football players on a team.

11. Identify the following measures as either quantitative or qualitative:

    a. The genders of the first 40 newborns in a hospital one year.
    b. The natural hair color of 20 randomly selected fashion models.
    c. The ages of 20 randomly selected fashion models.
    d. The fuel economy in miles per gallon of 20 new cars purchased last month.
    e. The political affiliation of 500 randomly selected voters.

12. A researcher wishes to estimate the average amount spent per person by visitors to a theme park. He takes a random sample of forty visitors and obtains an average of $28 per person.

a. What is the population of interest?
b. What is the parameter of interest?
c. Based on this sample, do we know the average amount spent per person by visitors to the park? Explain fully.

13. A researcher wishes to estimate the average weight of newborns in South America in the last five years. He takes a random sample of 235 newborns and obtains an average of 3.27 kilograms.

 a. What is the population of interest?
 b. What is the parameter of interest?
 c. Based on this sample, do we know the average weight of newborns in South America? Explain fully.

14. A researcher wishes to estimate the proportion of all adults who own a cell phone. He takes a random sample of 1,572 adults; 1,298 of them own a cell phone, hence 1298⁄1572 ≈ .83 or about 83% own a cell phone.

 a. What is the population of interest?
 b. What is the parameter of interest?
 c. What is the statistic involved?
 d. Based on this sample, do we know the proportion of all adults who own a cell phone? Explain fully.

15. A sociologist wishes to estimate the proportion of all adults in a certain region who have never married. In a random sample of 1,320 adults, 145 have never married, hence 145⁄1320 ≈ .11 or about 11% have never married.

 a. What is the population of interest?
 b. What is the parameter of interest?
 c. What is the statistic involved?
 d. Based on this sample, do we know the proportion of all adults who have never married? Explain fully.

16.
 a. What must be true of a sample if it is to give a reliable estimate of the value of a particular population parameter?
 b. What must be true of a sample if it is to give *certain* knowledge of the value of a particular population parameter?

ANSWERS

1. A population is the total collection of objects that are of interest in a statistical study.

3. A sample, being a subset, is typically smaller than the population. In a statistical study, all elements of a sample are available for observation, which is not typically the case for a population.

5. A parameter is a value describing a characteristic of a population. In a statistical study the value of a parameter is typically unknown.

7. All currently registered students at a particular college form a population. Two population characteristics of interest could be the average GPA and the proportion of students over 23 years.

9.
   a. Population.
   b. Sample.
   c. Population.
   d. Sample.
   e. Sample.

11.
   a. Qualitative.
   b. Qualitative.
   c. Quantitative.
   d. Quantitative.
   e. Qualitative.

13.
   a. All newborn babies in South America in the last five years.
   b. The average birth weight of all newborn babies in South America in the last five years.
   c. No, not exactly, but we know the approximate value of the average.

15.
   a. All adults in the region.
   b. The proportion of the adults in the region who have never married.
   c. The proportion computed from the sample, 0.1.
   d. No, not exactly, but we know the approximate value of the proportion.

1.2 Overview

LEARNING OBJECTIVE

1. To obtain an overview of the material in the text.

The example we have given in the first section seems fairly simple, but there are some significant problems that it illustrates. We have supposed that the 200 cars of the sample had an average value of $8,357 (a number that is precisely known), and concluded that the population has an average of about the same amount, although its precise value is still unknown. What would happen if someone were to take another sample of exactly the same size from exactly the same population? Would he get the same sample average as we did, $8,357? Almost surely not. In fact, if the investigator who took the second sample were to report precisely the same value, we would immediately become suspicious of his result. The sample average is an example of what is called a *random variable*: a number that varies from trial to trial of an experiment (in this case, from sample to sample), and does so in a way that cannot be predicted precisely. Random variables will be a central object of study for us, beginning in Chapter 4 "Discrete Random Variables".

Another issue that arises is that different samples have different levels of reliability. We have supposed that our sample of size 200 had an average of $8,357. If a sample of size 1,000 yielded an average value of $7,832, then we would naturally regard this latter number as likely to be a better estimate of the average value of all cars. How can this be expressed? An important idea that we will develop in Chapter 7 "Estimation" is that of the *confidence interval*: from the data we will construct an interval of values so that the process has a certain chance, say a 95% chance, of generating an interval that contains the actual population average. Thus instead of reporting a single estimate, $8,357, for the population mean, we would say that we are 95% certain that the true average is within $100 of our sample mean, that is, between $8,257 and $8,457, the number $100 having been computed from the sample data just like the sample mean $8,357 was. This will automatically indicate the reliability of the sample, since to obtain the same chance of containing the unknown parameter a large sample will typically produce a shorter interval than a small one will. But unless we perform a census, we can never be completely sure of the true average value of the population; the best that we can do is to make statements of *probability*, an important concept that we will begin to study formally in Chapter 3 "Basic Concepts of Probability".

Sampling may be done not only to estimate a population parameter, but to test a claim that is made about that parameter. Suppose a food package asserts that the amount of sugar in one serving of the product is 14 grams. A consumer group might suspect that it is more. How would they test the competing claims about the amount of sugar, 14 grams versus more than 14 grams? They might take a random sample of perhaps 20 food packages, measure the amount of sugar in one serving of each one, and average those amounts. They are not interested in the true amount of sugar in one serving in itself; their interest is simply whether the claim about the true amount is accurate. Stated another way, they are sampling not in order to estimate the average amount of sugar in one serving, but to see whether that amount, whatever it may be, is larger than 14 grams. Again because one can have certain knowledge only by taking a census, ideas of probability enter into the analysis. We will examine tests of hypotheses beginning in Chapter 8 "Testing Hypotheses".

Several times in this introduction we have used the term “random sample.” Generally the value of our data is only as good as the sample that produced it. For example, suppose we wish to estimate the proportion of all students at a large university who are females, which we denote by *p*. If we select 50 students at random and 27 of them are female, then a natural estimate is $p \approx \hat{p} = 27 \big/ 50 = 0.54$ or 54%. How much confidence we can place in this estimate depends not only on the size of the sample, but on its quality, whether or not it is truly random, or at least truly representative of the whole population. If all 50 students in our sample were drawn from a College of Nursing, then the proportion of female students in the sample is likely higher than that of the entire campus. If all 50 students were selected from a College of Engineering Sciences, then the proportion of students in the entire student body who are females could be underestimated. In either case, the estimate would be distorted or biased. In statistical practice an unbiased sampling scheme is important but in most cases not easy to produce. For this introductory course we will assume that all samples are either random or at least representative.

KEY TAKEAWAY

- Statistics computed from samples vary randomly from sample to sample. Conclusions made about population parameters are statements of probability.

1.3 Presentation of Data

LEARNING OBJECTIVE

1. To learn two ways that data will be presented in the text.

In this book we will use two formats for presenting data sets. The first is a **data list**[12], which is an explicit listing of all the individual measurements, either as a display with space between the individual measurements, or in set notation with individual measurements separated by commas.

EXAMPLE 1

The data obtained by measuring the age of 21 randomly selected students enrolled in freshman courses at a university could be presented as the data list

$$\begin{array}{ccccccccccc} 18 & 18 & 19 & 19 & 19 & 18 & 22 & 20 & 18 & 18 & 17 \\ 19 & 18 & 24 & 18 & 20 & 18 & 21 & 20 & 17 & 19 & \end{array}$$

or in set notation as

$$\{18,18,19,19,19,18,22,20,18,18,17,19,18,24,18,20,18,21,20,17,19\}$$

A data set can also be presented by means of a **data frequency table**[13], a table in which each *distinct* value x is listed in the first row and its **frequency**[14] f, which is the number of times the value x appears in the data set, is listed below it in the second row.

[12] An explicit listing of all the individual measurements made on a sample.

[13] A table listing each distinct value x and its frequency f.

[14] How often a value x appears in a data set.

EXAMPLE 2

The data set of the previous example is represented by the data frequency table

x	17	18	19	20	21	22	24
f	2	8	5	3	1	1	1

The data frequency table is especially convenient when data sets are large and the number of distinct values is not too large.

KEY TAKEAWAY

- Data sets can be presented either by listing all the elements or by giving a table of values and frequencies.

EXERCISES

1. List all the measurements for the data set represented by the following data frequency table.

x	31	32	33	34	35
f	1	5	6	4	2

2. List all the measurements for the data set represented by the following data frequency table.

x	97	98	99	100	101	102	103	105
f	7	5	3	4	2	2	1	1

3. Construct the data frequency table for the following data set.

$$\begin{array}{cccccc} 22 & 25 & 22 & 27 & 24 & 23 \\ 26 & 24 & 22 & 24 & 26 & \end{array}$$

4. Construct the data frequency table for the following data set.

$$\{1,5,2,3,5,1,4,4,4,3,2,5,1,3,2,$$
$$1,1,1,2\}$$

ANSWERS

1. {31,32,32,32,32,32,33,33,33,33,33,33,34,34,34,34,35,35}.

3.

x	22	23	24	25	26	27
f	3	1	3	1	2	1

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